Markov Point Process for Multiple Object Detection

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Outline

Motivation

The different issues :

- objects
- reference measure
- prior
- data term
- optimization

Some results
Motivation: from context to geometry

1) High resolution data: the object geometry is an important source of information
2) The pixel scale does not contain the main information
Motivation: from context to geometry

1) Consider prior information (Bayesian approach, Markov Random Fields, interactions)

2) Embed geometric information (graph of objects)

3) Modeling the scene structure (interactions between objects, unknown number of objects)

4) Need algorithms for simulating, optimizing the models

Marked point processes
Motivation: a scene as a collection of objects
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Motivation: a scene as a collection of objects
The configuration space

« Simple » parametric shapes: \[ S = \{ s = (x, m), x \in K, m \in M \} \]

\[ \Omega_n = \left\{ \{s_1, \ldots, s_n\}, s_i \in S \right\} \quad \Omega_0 = \emptyset \]

\[ \Omega = \bigcup_{n} \Omega_n \]
The reference measure

Usually the Poisson measure:

$$
\pi_\nu(B) = e^{-\nu(x)} \left( 1_{\emptyset \in B} + \sum_{n=0}^{\infty} \frac{\pi_{\nu_n}(B)}{n!} \right)
$$

$$
\pi_{\nu_n}(B) = \int \cdots \int 1_{\{x_1, \ldots, x_n \in B\}} \nu(dx_1) \cdots \nu(dx_n)
$$

Intensity measure: uniform or not

$$\nu(A) \int_A \lambda(x) dx$$

NDVI MAP
The density

The model is defined by a density (usually un-normalised) w.r.t the reference measure:

\[ f : \Omega \rightarrow [0, \infty[, \int_{\Omega} f(x) d\pi_{\nu}(x) < \infty \]

Mimicking the Bayesian approach:

\[ f(x) = g(x) h_I(x) \]

Prior \hspace{2cm} Data (I) term
The prior

Overlap penalization (pairwise interaction):

\[ g(x) = \prod_{i \sim j} \varphi(x_i, x_j) \quad \varphi(x_i, x_j) = \Phi \left( \frac{|s_i \cap s_j|}{\min(|s_i|, |s_j|)} \right) \]

Alignment:

\[ g(x) = \prod_{i \sim j \sim k} \varphi(x_i, x_j, x_k) \quad \varphi(x_i, x_j, x_k) = \Phi \left( |\pi - \theta_{i,j,k}| \right) \]

Overlap penalization:

\[ g(x) = \prod_{i} \varphi(x_i, x_j, j \sim i) \quad \varphi(x_i, x_j, j \sim i) = \Phi \left( \max_{j \sim i} \left( \frac{|s_i \cap s_j|}{\min(|s_i|, |s_j|)} \right) \right) \]

And many more …
The data term

Bayesian approach: likelihood

\[ h_Y(x) = \prod_{i \text{ inside objects}} l_{\text{object}}(y_i) \prod_{i \text{ outside objects}} l_{\text{background}}(y_i) \]

Distance between interior and exterior

\[ h_Y(x) = \prod_{\text{objects}} \text{dissimilarity}(\text{red pixels, blue pixels}) \]

Geometrical consistency

\[ h_Y(x) = \prod_{\text{objects}} \exp - U(\omega) \quad U(\omega) = \frac{1}{|\partial \omega|} \int \left( \frac{\nabla Y(u)}{\sqrt{||\nabla Y(u)||^2 + \epsilon^2}} , n(u) \right) du \]

\[ U_d(\omega,Y) = \psi(U(\omega),t) \]

And many more …
Optimization : RJMCMC

- Initialize the temperature $T$ and the configuration $x$ (empty set).
- Choose a proposition kernel $Q_m(x,.)$ with probability $p_m(x)$, or let the configuration unchanged probability $1-\sum_m p_m(x)$.
- Sample $x'$ according to the chosen kernel.
- Compute the acceptation ratio:

$$R_m(x, x') = \frac{D_m(x', x)}{D_m(x, x')} = \frac{(h(x'))^{\frac{1}{T}} \pi(dx') Q_m(x', dx)}{(h(x))^{\frac{1}{T}} \pi(dx) Q_m(x, dx')}$$

- With probability $\alpha = \min(1, R_m)$ set $x_{t+1} = x'$, else reject the proposition : $x_{t+1} = x$.

Some perturbation kernels (proposal)
- Adding an object
- Removing an object
- Modifying an object (translation, rotation, dilation)
- Merging/Splitting objects
Optimization : RJMCMC

Pros :
- Generality
- Choice for kernels
- Convergence to the global optimum

Cons :
- Rejection
- Simulated annealing scheme (parameters setting)
- Kernels usually involve one or two objects
Optimization : Multiple births and deaths

Goals:
- Avoid rejection
- Consider several objects at once

Idea:
- Extend Langevin’s dynamics (Stochastic Differential Equation: diffusion process)
Optimization: Multiple births and deaths

Consider: \[ f(x) = \exp -\beta E(x) \]

A birth intensity consisting in adding an object \( u \) to the configuration \( x \):

\[ b(x,u)du = zdu \text{ if } u \in D(x), \text{ where } D(x) = \bigcup_{v \in x} B_v(\varepsilon) \cap K \]

A death intensity consisting in removing an object \( u \) from the configuration \( x \):

\[ d(x,u) = \exp \beta \left[ E(x) - E(x/u) \right] \text{ if } u \in x, \]

Detailed balance condition holds.
Optimization: Multiple births and deaths

A New Approximation Process

Markov Chain: $T_{\beta, \delta} (m), m = \left\lfloor \frac{t}{\delta} \right\rfloor = 0, 1, 2, \cdots$ (discretization of time)

$x_{n+1} = x_1 \cup x_2, \ x_1 \subseteq x_n, \ x_1 \cap x_2 = \emptyset$

$x_2$: Poisson law (with intensity $z$) distributed

Birth transition:

$\begin{cases} 
z\delta \Delta u, & \text{if } x \rightarrow x \cup \{u\} \\
1 - z\delta \Delta u, & \text{if } x \rightarrow x \ (\text{no birth in } \Delta y) 
\end{cases}$

Death transition:

$\begin{cases} 
\frac{\delta \exp[E(x) - E(x/u)]}{1 + \delta \exp[E(x) - E(x/u)]}, & \text{if } x \rightarrow x/u \\
1 & \text{if } x \rightarrow x \ (x \text{ survives}) 
\end{cases}$
Optimization: Multiple births and deaths

A New Algorithm

1) Precomputing of the data term / birth map

2) Repeat:

2.1) Birth:
   For each pixel, add an object with probability:
   \[ \delta B(E_d(u)) \]

2.2) Sort the objects with respect to their data term value

2.3) Death:
   For each object \( u \) taken in the list order, remove it with probability:
   \[ \frac{\delta \exp[\beta(E(x) - E(x/u))]}{1 + \delta \exp[\beta(E(x) - E(x/u))]} \]
Optimization: Multiple births and deaths

**Theorem:** When $N \to \infty$, $\beta \to \infty$, $\delta \to 0$, convergence toward the configuration minimizing the energy.

**Pros:**
- No rejection in the birth step
- Birth does not depend on the temperature
- Convergence to the global optimum

**Cons:**
- Only births and deaths kernels
Optimization : Multiple births and cut

Idea: Combine multiples births and deaths with graphcut techniques

- **Generate a first configuration** $x_0$ **of non overlapping objects** and iterate the following steps:
  - **Birth:**
    - generate a new configuration of non overlapping objects $x'$
  - **Death:**
    - $x_{n+1}$ is defined by $\text{Cut}(x_n \cup x')$
Optimization: Multiple births and cut

Pros:
- No rejection in the birth step
- No cooling schedule

Cons:
- Only births and deaths
- No proof of convergence
Result: Trees counting
Result: Trees counting
Result: Trees counting
Result: Flamingos counting

Estimation of the size of a colony in Turkey (2004):

卓 3682 detected flamingos (Tour du Valat: 3684 flamingos)
Result: Flamingos counting

Estimation of the size of a colony in Mauritania:

🌟 Very high density
Result: Flamingos counting

Estimation of the size of a colony in Mauritania:

✿ 14595 detected flamingos (Tour du Valat: 13650 flamingos)
Result: Cells counting
Result: Cells counting
Result: Vesicule (co)localization
Pros:
- General framework / Numerous application
- Embed strong geometric constraint

Future work:
- Parallelism
- Parameter estimation
- Open source software